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convergent sequences of points. Dedekind's axiom is to the effect that to the irrational number defined by the class of numerical sequences corresponds one and only one point, namely, that which is the limit point of the corresponding sequences of points. This, it seems to me, is the adequate enlargement of our idea of "number of times contained" so that we can speak as definitely and rigorously of one segment being contained $\sqrt{2}$ times in another segment as we do when we say that one is contained 1.4 times in another. In brief, *for the theory of measurement in elementary geometry the limit process has been applied once for all in the correlation of real numbers with the points on a line.*

The present difficulties are encountered because we are allowing ourselves to be confined to the number system that Euclid knew—and which was adequate for his method of treating proportion—while trying to treat proportion by arithmetic, which he did not do.

According to the foregoing argument, the results of modern research justify us in regarding every two segments as commensurable and, consequently, all theorems on proportional lines, angles and areas will be proved as they now are in the conventional "commensurable case."

It may be asked, however, what must be taught to high school students for the Dedekind axiom and how a revised proof would run. The usual distinction between commensurable and incommensurable would be replaced by the explicit assumption: *For every two like magnitudes there exists a number that denotes the number of times the one is contained in the other.*

Not only is the equivalent of this axiom at the foundation of analysis but it is the very assumption that seems reasonable to the student. Moreover, we do use it. Let us, therefore, make it explicit.

In secondary school work irrationals are introduced in arithmetic and some use is made of them in the first course in algebra, Since we proceed as if our students had the whole real number system, is it unreasonable to apply it to geometry? Possibly our methods of teaching irrational numbers may need revision, but that is a question for arithmetic and algebra rather than for geometry.

With the foundations for a simple and complete theory of measurement laid in the needed assumption, the proofs for the three important theorems would be simplified. In fact, they would be virtually the same as in the ordinary "commensurable case" except that the numbers used would represent any real numbers instead of merely integers.

The case of limits in the measurement of the circle is different, since in that case the difficulty is not with the limit (whose existence is postulated) but with the setting up of a numerical sequence which will evaluate that limit approximately.

COMMUNICATION.

To the Editor of the Monthly: It seems that a criticism which I made in reviewing Dowling and Turneaure's *Analytic Geometry* in the March number of the Monthly was not very clear, since Professor Dowling states in the May number that the proof which I said was lacking is given on page 58, whereas I find there no answer to the point which I wished to raise. Inasmuch as the criticism applies also to another recent text (SMITH and GALE: *New Analytic Geometry*) I will ask space for a more detailed statement of my point.

By definition, the equation of the locus of a point satisfying a given condition is an equation in the variables x and y representing coordinates such that, (1) if a point $P(x, y)$ is on the locus it satisfies the equation, and conversely, (2) if a point $P(x, y)$ satisfies the equation it is on the locus.[1]

In deriving the equation of a straight line, Dowling and Turneaure proceed as follows on pages 57, 58:

> Choose any point $P(x, y)$ on the line joining P_1 and P_2 (given points). Then, slope of segment P_1P = slope of segment P_1P_2. But (Art. 11), slope of $P_1P = (y - y_1)/(x - x_1)$, and slope of $P_1P_2 = (y_2 - y_1)/(x_2 - x_1)$. Therefore $(y - y_1)/(x - x_1) = (y_2 - y_1)/(x_2 - x_1)$. (1)

Immediately following this, the authors change this equation into the slope-point form, the slope form, and the intercept form. They then say, "Equations (1), (2), (3), and (4) are all standard forms of the equation of a straight line." My criticism is that they have not shown that any of these equations is the equation of a straight line. They have shown that equation (1) satisfies the first requirement of the definition of the equation of a locus (*i. e.*, if $P(x, y)$ is on the line it satisfies the equation) but they have not proved the "and conversely" part of the definition to be satisfied (*i. e.*, they have not shown that if a point $P(x, y)$ satisfies the equation it lies on the locus). A complete proof of the "and conversely" is as difficult as the part of the proof which they have given, and it should not be omitted.

Following the last statement quoted, the authors name the several standard forms, and then from these equations conclude that, "the equation of a straight line is of first degree in the variables x and y." They then say, "Conversely, it may be shown that any equation of the first degree in the variables x and y is the equation of a straight line." In the proof of this theorem they assume that they have proved that $y = mx + b$ is the equation of a straight line. Obviously then the proof of this "conversely" cannot nullify the criticism I have made above.

Smith and Gale have made the same omission on page 58 of their text.

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[1] This is essentially the way in which the definition is given in both texts. See page 55 of the former and page 32 of the latter.